point on the curves. This means that, for example, for $q = 10^9$ W/m^2 the flow will be stable relative to infinitesimal planar perturbations with an inclination of less than 14°. But, for $q > 2.01 \cdot 10^9$ W/m^2 (for $q = 2.01 \cdot 10^9$ W/m^2 the neutral curve degenerates into a point), the flow will be stable up to a vertical orientation of the film.

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CALCULATION OF FLOWS OF MELT IN AN AMPULE

V. V. Kuznetsov

UDC 532.526.2

The problem of determining the dopant distribution in crystallization under conditions of reduced gravitational force is of current interest. The physical characteristics of such processes are examined in [1-3]. To solve this problem, one must know the flow velocity field of the melt. Schemes for solving this problem, primarily for moderate Reynolds and Marangoni numbers, are proposed in a number of papers [4, 5].

In this paper we propose an asymptotic scheme of stationary thermocapillary convection in a cylindrical ampule with large Reynolds and Marangoni numbers; this situation is realized in the presence of very high temperature differentials along the lateral wall of the ampule and low viscosity of the melt. The flow consists of a collection of Prandtl and Marangoni boundary layers, which join to the core of the flow. The axisymmetrical circulating flow in the core is calculated using the Prandtl–Batchelor scheme. The thermocapillary convection of the melt in the ampule is calculated using this scheme.

1. We are examining the problem of determining the thermocapillary convection velocity field of the melt in a cylindrical ampule with directed crystallization in the absence of gravity. The region of flow is illustrated in Fig. 1. The volume compression of semiconducting materials with melting gives rise to voids in the ampule, which are assumed to be distributed along the lateral wall of the ampule. The flow is assumed to be laminar, stationary, and axisymmetrical. The assumption of stationariness is explained by the fact that the time of crystallization of the entire ampule usually is several hours, so that the velocity of the crystallization front is of the order of 10^{-4} cm/sec, which is much lower than the velocity of thermocapillary convection with a very large temperature drop along the ampule.

Under the assumptions made above, the flow is described by the system of Navier–Stokes equations

$$uu_r + wu_z = -\frac{p_r}{\rho} + \nu\left(u_{rr} + \frac{1}{r}u_r - \frac{1}{r^2}u + u_{zz}\right), \quad (1.1)$$
$$uw_r + ww_z = -\frac{p_z}{\rho} + \nu\left(w_{rr} + \frac{1}{r}w_r + w_{zz}\right),$$
$$u_r + \frac{1}{r}u + w_z = 0,$$

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where u and w are the transverse and longitudinal components of the velocity vector. The conditions for attachment are imposed at the crystallization front z = L and at the bottom of the ampule z = 0

$$u = w = 0, \tag{1.2}$$

and the condition

$$p = p_0 + 2\sigma K + 2\rho\nu\mathbf{n}\cdot D\cdot\mathbf{n},\ \ 2\rho\nu\mathbf{s}\cdot D\cdot\mathbf{n} = \partial\sigma/\partial s,\ \ wH_z = u \tag{1.3}$$

is imposed at the free lateral surface r = H(z). Here **s** and **n** are the tangent vector and the outer normal to the surface r = H(z); D is the tensor for the deformation velocities of the melt; ρ and ν are the density and coefficient of kinematic viscosity; K is the average curvature of the surface r = H(z); the coefficient of surface tension $\sigma = \sigma_0 - \sigma_T(T - T_0)$, where $\sigma_T = d\sigma/dT$ = const. The temperature along the lateral surface of the melt is assumed to be a fixed function of the coordinate z.

2. To solve the problem (1.1)-(1.3), we single out the characteristic zone in the region occupied by the melt. It is well known that, for sufficiently large temperature gradients along the free surface, it is possible to single out the Marangoni boundary layer, whose mathematical model is given in [6]. In addition, we shall single out the boundary layers near the crystallization front and at the bottom of the ampule. All three boundary layers are joined to the main core of the flow of the melt. We assume that the free surface of the melt r = H(z) is weakly curved, i.e., $(dH/dz)^2 \ll 1$. This assumption agrees with the fact that, in many semiconducting materials, for example germanium, the angles of touching and of growth are close to π and π/2, respectively. Under the assumptions made, the second of the conditions (1.3) permits estimating the characteristic velocity V of the melt in the Marangoni boundary layer. This condition can be approximately written in the form $\rho\nu w_r = |\sigma_T| dT/dz$, where the thickness of the Marangoni boundary layer $w_r = V/\delta$, $dT/dz = \Delta T/L$ is the temperature drop along the lateral surface, Reynolds number Re = LV/ν, we obtain for the characteristic velocity the value

$$V = (|\sigma_T|\Delta T/\rho\nu)^{2/3}(\nu L)^{1/3}. \tag{2.1}$$

The intensity of the Marangoni flow can be estimated with the help of the Marangoni number $M = |\sigma_T|\Delta TL/\rho\nu^2$, which represents the ratio of thermocapillary and viscous forces. Setting the temperature drop equal to 100° and choosing the value of the material constants corresponding to fused germanium, we obtained a value of M of the order of 10^6. Equation (2.1) gives a characteristic value of the velocity V of the order of 5 cm/sec.

Let us assume that the principal core of the flow of the melt, to which the flows of all three boundary layers join, represents a limiting flow in the limit Re → ∞ with closed streamlines. We introduce the dimensionless quantities using the formulas z = LZ, r = aR, u = UV, w = WV, where a is the radius of the ampule. As shown in [7], the only nonvanishing component of the vortex η in this case satisfies the relation $\eta = C_1R$, where C_1 is some con-

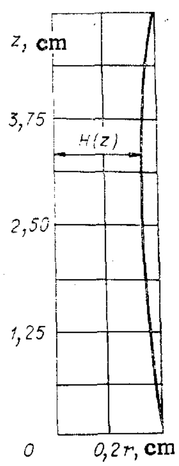

Fig. 1

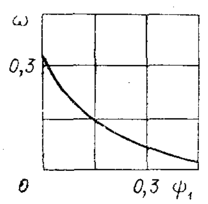

Fig. 2

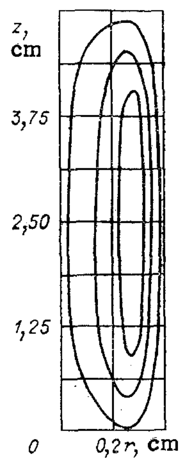

Fig. 3

stant. Introducing the stream function ψ, so that $U = (1/R)\partial\psi/\partial Z$, $W = -(1/\lambda R)\partial\psi/\partial R$, we arrive at the problem of finding the stream function:

$$\lambda^2 \frac{\partial^2\psi}{\partial Z^2} + \frac{\partial^2\psi}{\partial R^2} - \frac{1}{R}\frac{\partial\psi}{\partial R} = CR^2, \quad \lambda = a/L, \quad C = C_1 a^2/V \tag{2.2}$$

with the boundary condition

$$\psi|_\Gamma = 0, \text{where } \Gamma \text{ is the boundary of the square} \leqslant R \leqslant 1, \; 0 \leqslant Z \leqslant 1. \tag{2.3}$$

The solution of problem (2.2), (2.3) is given by the formula

$$\psi = C \sum_{k=1}^{\infty} \frac{2RJ_1(\lambda_k R)}{J_2(\lambda_k)\lambda_k^2} \left(\frac{\exp(\lambda_k (Z-1)/\lambda) + \exp(-\lambda_k Z/\lambda)}{1 + \exp(-\lambda_k/\lambda)} - 1 \right), \tag{2.4}$$

where J_1 and J_2 are Bessel functions of the first kind of order 1 and 2, respectively; the numbers λ_k ($k = 1, 2, \ldots$) are the roots of the function J_1. The constant C, on which the solution of the problem (2.2), (2.3) depends, will be determined numerically below.

3. In dimensionless variables, the problem for Marangoni's boundary layer near the free lateral surface is written in the form

$$u_1 \frac{\partial w_1}{\partial r_1} + w_1 \frac{\partial w_1}{\partial z_1} = -\frac{\partial p_1}{\partial z_1} + \frac{\partial^2 w_1}{\partial r_1^2}, \tag{3.1}$$
$$\partial u_1/\partial r_1 + \partial w_1/\partial z_1 = 0, \; \partial p_1/\partial r_1 = 0, \; 0 < z_1 < 1, \; 0 < r_1 < \infty$$

with the boundary conditions

$$\partial w_1/\partial r_1 = d\theta/dz_1,$$
$$p_1 = P_0 + P_1 h, \; P_0 = \sigma/\rho V^2 a, \; P_1 = \sigma\delta/\rho V^2 a^2, \tag{3.2}$$
$$w_1 \partial h/\partial z_1 = u_1 \text{ at } r_1 = h(z_1), \; w_1 \to W(1, z_1) \text{ as } r_1 \to \infty.$$

The problem (3.1), (3.2) can be formally obtained by introducing the dimensionless variables r_1, z_1, u_1, w_1, p_1, h, and θ defined as $r = a - \delta r_1$, $H = a - \delta h$, $u = -\varepsilon V u_1$, $w = V w_1$, $z = L z_1$, $p = p_0 + \rho V^2 p_1$, $T = T_0 + \Delta T\theta$, by substituting into problem (1.1)-(1.3) and retaining first-order terms. Here, $\varepsilon = 1/\sqrt{Re}$. The pressure gradient is found from Bernoulli's integral, if it is applied to the core of the flows [7]. Thus, $\partial p_1/\partial z_1 = -W(1, z_1)\, W(1, z_1)/\partial z_1$.

A characteristic feature of the problem (3.1), (3.2) is that in contrast to the problem (1.1)-(1.3) the position of the free boundary can be found independently of the velocity components u_1, w_1, since the second condition (3.2) can be used to find the function $h(z_1)$. Physically, this means that in the boundary layer approximation capillary forces play the main role in determining the form of the free surface. Let us write the problem (3.1), (3.2) in Mies's variables z_1 and ψ_1:

$$\frac{\partial\omega}{\partial z_1} = \sqrt{\omega}\frac{\partial^2\omega}{\partial\psi_1^2} - 2\frac{\partial p_1}{\partial z_1}; \tag{3.3}$$

$$\partial\omega/\partial\psi_1 = 2d\theta/dz_1 \equiv A(z_1) \text{ at } \psi_1 = 0; \tag{3.4}$$

$$\omega \to W^2(1, z_1) \text{ as } \psi_1 \to \infty, \tag{3.5}$$

where

$$\omega = w_1^2; \; u_1 = \partial\psi_1/\partial z_1; \; w_1 = -\partial\psi_1/\partial r_1.$$

We single out the boundary layers at the crystallization front and at the bottom of the ampule. Introducing the dimensionless quantities r_0, z_0, p_1, u_0, w_0, defined as $r = a(1 - r_0)$, $z = L - \varepsilon a z_0$, $u = -Vu_0$, $w = -\varepsilon V w_0$, and retaining in the system of equations (1.1), after substituting first-order terms, we obtain

$$\frac{\partial u_0}{\partial r_0} - \frac{u_0}{1 - r_0} + \frac{\partial w_0}{\partial z_0} = 0,$$

$$u_0 \frac{\partial u_0}{\partial r_0} + w_0 \frac{\partial u_0}{\partial z_0} = -\frac{\partial p_1}{\partial r_0} + \frac{\partial^2 u_0}{\partial z_0^2}, \quad \frac{\partial p_1}{\partial z_0} = 0$$

with the boundary conditions

$$u_0 = w_0 = 0 \text{ at } z_0 = 0,\ u_0 \to U(1 - r_0,\ 1) \text{ as } z_0 \to \infty.$$

From Bernoulli's integral $\partial p_1/\partial r_0 = -U(1 - r_0, 1)\partial U(1 - r_0, 1)/\partial r_0$. The problem obtained has a singularity as $r_0 \to 1$. Using (2.4), it is evident that $\partial p_1/\partial r_0 = O(1 - r_0)$. If we assume that $\partial p_1/\partial r_0 = \alpha^2(1 - r_0)$, then the self-similar solution can have the form

$$u_0 = -\alpha(1 - r_0)d\varphi(\xi)/d\xi,\ w_0 = -\sqrt{2\alpha}\varphi(\xi),\ \xi = z_0/\sqrt{2\alpha}.$$

To find the function φ we obtain the problem

$$d^3\varphi/d\xi^3 + \varphi d^2\varphi/d\xi^2 + (1/2)(1 - (d\varphi/d\xi)^2) = 0, \tag{3.6}$$
$$\varphi = d\varphi/d\xi = 0 \text{ at } \xi = 0,\ d\varphi/d\xi \to 1 \text{ as } \xi \to \infty.$$

As demonstrated in [8], problem (3.6) has a unique solution. It may thus be expected that as $r_0 \to 1$, $u_0 = O(1 - r_0)$, $w_0 = O(1)$. Introducing Mies's variables q, r_0, and ψ_0, defined as $\partial\psi_0/\partial z_0 = (1 - r_0)u_0$, $\partial\psi_0/\partial r_0 = -(1 - r_0)w_0$, we obtain the problem

$$\frac{\partial q}{\partial r_0} = (1 - r_0)^2 \sqrt{q}\,\frac{\partial^2 q}{\partial \psi_0^2} - 2\frac{\partial p_1}{\partial r_0} \tag{3.7}$$

with the boundary conditions

$$q = 0 \text{ at } \psi_0 = 0,\ q \to U^2(1 - r_0,\ 1) \text{ as } \psi_0 \to \infty. \tag{3.8}$$

Performing an analogous expansion, we obtain the problem for finding the velocity in the boundary layer at the bottom of the ampule. Setting $r = ar_2$, $z = \varepsilon a z_2$, $u = Vu_2$, $w = \varepsilon V w_2$, $p = \rho V^2 p_2$ and, in addition, $r_2u_2 = \partial\psi_2/\partial z_2$, $r_2w_2 = -\partial\psi_2/\partial r_2$, $v = u^2{}_2$, we obtain the problem

$$\partial v/\partial r_2 = r_2^2 \sqrt{v}\,\partial^2 v/\partial\psi_2^2 - 2\partial p_2/\partial r_2 \tag{3.9}$$

with the boundary conditions

$$v = 0 \text{ at } \psi_2 = 0,\ v \to U^2(r_2,\ 0) \text{ as } \psi_2 \to \infty. \tag{3.10}$$

4. All of the problems obtained above are determined if the constant C from (2.4) is known. To determine C we note that for a stationary liquid volume Ω with boundary Σ, the following energy identity holds:

$$\int_\Sigma \mathbf{t}\cdot\mathbf{v}\,d\Sigma = 2\rho\nu \int_\Omega D{:}D\,d\Omega, \tag{4.1}$$

where $\mathbf{t}$ is the stress vector; $\mathbf{v}$ is the velocity vector of the liquid at the boundary Σ; D is the tensor of deformation velocities. Using the boundary conditions (1.2) and (1.3), we write the first term of the identity (4.1), denoted by I, in the form

$$I = \frac{1}{\varepsilon}\pi\rho\nu V^2 \int_0^1 \sqrt{\omega}\,|_{\psi_1=0} A(z_1)\,dz_1.$$

We shall calculate the dissipation of energy over all separated zones of the liquid volume. In the core of the flow the dissipative functions $\chi_3 = D{:}D$ will equal

$$\chi_3 = V^2\left\{\left(\frac{\partial U}{\partial R}\right)^2 + \left(\frac{U}{R}\right)^2 + \left(\lambda\frac{\partial W}{\partial z}\right)^2 + \frac{1}{2}\left(\frac{\partial W}{\partial R} + \lambda\frac{\partial U}{\partial Z}\right)^2\right\}\Big/a^2 = V^2\left\{\left(\frac{\partial U}{\partial R} + \frac{U}{R} + \lambda\frac{\partial W}{\partial Z}\right)^2 - 2\left(\lambda\frac{\partial U}{\partial R}\frac{\partial W}{\partial Z} + \frac{U}{R}\frac{\partial U}{\partial R} + \right.\right.$$
$$\left.\left. + \lambda\frac{U}{R}\frac{\partial W}{\partial Z}\right) + \frac{1}{2}\left(\frac{\partial W}{\partial R} - \lambda\frac{\partial U}{\partial Z}\right)^2 + 2\lambda\frac{\partial U}{\partial Z}\frac{\partial W}{\partial R}\right\}\Big/a^2 = V^2\left(\frac{C^2R^2}{2\lambda} + \frac{2U^2}{R^2} + 2\lambda\frac{\partial W}{\partial R}\frac{\partial U}{\partial Z} - 2\lambda\frac{\partial U}{\partial R}\frac{\partial W}{\partial Z}\right)\Big/a^2.$$

After integrating χ_3 over the region of the core we find that the energy dissipation in the core of the flow

$$I_3 = 4\pi\rho\nu V^2\left\{\frac{C^2}{6\lambda} + 2\int_0^1\int_0^1\left(\frac{U}{R}\right)^2 dZ\,dR\right\}.$$

By means of simple transformations we find that the dissipation of energy in the boundary layers at the crystallization front, at the bottom of the ampule, and opposite the free surface equal, respectively,

$$I_0 = \frac{1}{2\varepsilon}\pi\rho\nu V^2 \int_0^1\int_0^\infty \frac{(1-x)\,(\partial q\,(x,\,\psi)/\partial\psi)^2}{\sqrt{q}}\,d\psi dx,$$

$$I_2 = \frac{1}{2\varepsilon}\pi\rho\nu V^2 \int_0^1\int_0^\infty \frac{x\,(\partial v\,(x,\,\psi)/\partial\psi)^2}{\sqrt{v}}\,d\psi dx,$$

$$I_1 = \frac{1}{2\varepsilon}\pi\rho\nu V^2 \int_0^1\int_0^\infty \frac{(\partial\omega\,(x,\,\psi)/\partial\psi)^2}{\sqrt{\omega}}\,d\psi dx.$$

We transform the integral from the last equality, using (3.3) and (3.4),

$$\int_0^1\int_0^\infty (\omega_\psi)^2/\sqrt{\omega}\,d\psi dx = 2\int_0^1\int_0^\infty \left(\partial\left(\sqrt{\omega}\,\omega_\psi\right)/\partial\psi - \sqrt{\omega}\,\omega_{\psi\psi}\right)d\psi dx =$$

$$= 2\int_0^1 \sqrt{\omega}\,\omega_\psi\big|_{\psi=0}\,dx - 2\int_0^1\int_0^\infty \partial\,(\omega - W\,(1,\,x))/\partial x d\psi dx \;= 2\int_0^1 \sqrt{\omega}\,\big|_{\psi=0} A\;\;(x)\,dx - 2\int_0^\infty (\omega - W\,(1,\,x))\big|_0^1\,d\psi.$$

Substituting the relations obtained into (4.1), we obtain the equation for finding the constant C:

$$F\,(C) \equiv \frac{2\varepsilon C^2}{3\lambda} + 8\varepsilon\int_0^1\int_0^1 \left(\frac{U}{R}\right)^2 dR dZ - \int_0^\infty (\omega - W\,(1,\,x))\big|_0^1\,d\psi \;+ \frac{1}{2}\int_0^1\int_0^\infty \left\{\frac{(1-x)\,(q_\psi)^2}{\sqrt{q}} + \frac{x\,(v_\psi)^2}{\sqrt{v}}\right\} d\psi dx = 0. \quad (4.2)$$

The numerical calculation was performed using the following scheme: The constant C was initially set equal to C = 5, the flow in the core was found from formula (2.4), and then the problem (3.3)-(3.5), (3.7), (3.8), (3.9), and (3.10) was solved numerically, after which the left side of Eq. (4.2) was calculated and the value of the constant C was corrected by the method of halving the segment. After eight iterations, to within 10^{-3}, the process converged to the value C = 0.136. The material constants ρ, σ, σ_T, and ν were chosen to correspond to melted germanium with T = 937°C, the temperature along the side wall was assumed to be distributed parabolically with the apex of the parabola lying at the bottom of the ampule and the temperature drop ΔT = 100°. The dimensions of the ampule were chosen to be L = 5 cm and a = 0.4 cm. The small value of the constant C shows that the intense motion of the melt occurs only in a very thin layer (of the order of 10^{-2} of the radius) next to the free surface. Figure 2 shows the decrease of the square of the longitudinal velocity in the Marangoni boundary layer with distance from the free surface in a section of the ampule by a plane equidistant from the bottom of the ampule and the crystallization front. Figure 3 shows the stream lines in the core of the flow. It is evident that the bunchup at boundaries provides a posteriori justification for the use of the boundary-layer approximation.

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EXPLOSION OF A SPHERICAL CHARGE IN A MAGNETIC FIELD

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The need to study the interaction of detonation waves with a magnetic field arises in research involving many phenomena, for example nonstationary flows of cosmic matter, as well as in practicall applications, for example in creating explosive MHD generators. Several problems involving explosions, taking into account the effect of a magnetic field for the case of a point explosion, are formulated and solved in [1].

The problem of an explosion of a cylindrical charge of condensed explosive in a gas in the presence of an external magnetic field is examined in [2]. In this paper we study the analogous problem for a spherical charge. The main difference, from the mathematical point of view, between this problem and the preceding one lies in the fact that its solution depends on two spatial coordinates (r, z in a cylindrical coordinate system) and time t, i.e., the problem becomes two-dimensional. The scheme for the flow that arises is shown in Fig. 1, where 1 denotes the products of the detonation, 2 denotes the contact surface, 3 denotes the shock-compressed gas, and 4 denotes the shock wave.

The interaction with the magnetic field occurs as a result of the motion of the electrically conducting gas, heated up by the shock wave, across the force lines of the magnetic field. The flow will differ from the spherically symmetrical flow that occurs in the absence of the field. In particular, the form of the contact surface bounding the detonation products and the form of the shock waves arising in the surrounding gas will become gradually distorted, stretching out along the force lines of the magnetic field.

The problem was solved in the approximation of small magnetic Reynolds numbers R_m (in the calculations $R_m \leqslant 0.1$); in addition, the deformations of the initial magnetic field were ignored. In taking into account radiation losses, we also used the approximation of volume emission. The detonation products are assumed to be electrically nonconducting [3] and nonemitting. The detonation wave is initiated at the center of the charge. Right up to the moment that the wave emerges onto the surface of the charge, the solution is self-similar and can be found separately. Then, it is already necessary to solve the complete system of two-dimensional equations of magnetogasdynamics, which have the form

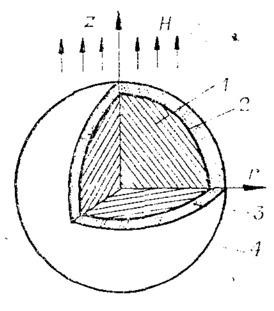

Fig. 1

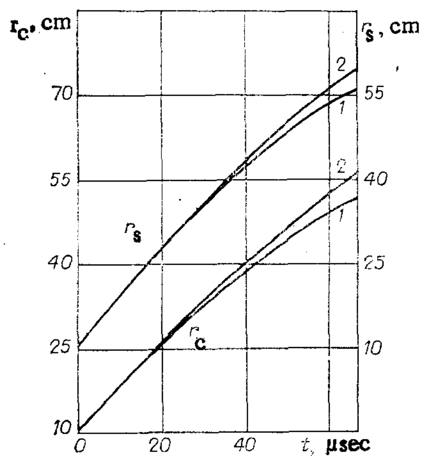

Fig. 2

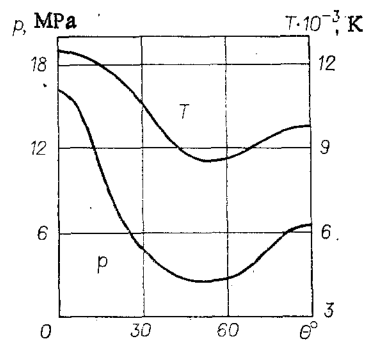

Fig. 3

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